This Technical Bulletin discusses the structural analysis of uniformly loaded floor slabs supported by Rammed Aggregate Pier® (RAP) soil reinforcing elements. RAP soil reinforcing elements are commonly used to support concrete floor slabs, eliminating the need for structural slabs supported on deep foundations or massive excavation and recompaition required for slab-on-grade construction. The piers reduce total and differential settlements because of their high strength and high stiffness characteristics. Because of the variation in pier stiffness compared to in-situ soil stiffness, however, the assumption of uniform sub-grade support is no longer valid. The dissimilar slab support conditions, consisting of high stiffness at the pier locations and relatively low stiffness between the piers, leads to the development of bending moments and shear stresses within the slabs under applied load.

This Technical Bulletin describes the result of a series of finite element analyses performed to quantify the bending moment and shear stress conditions that develop in relatively thin floor slabs supported by RAP elements. This bulletin provides design charts that may be used to estimate required concrete slab thickness for a floor slab with a uniformly distributed loading condition supported by RAP elements. The charts should be used with judgement, however, because it is recognized that a uniformly distributed loading analysis may not capture the critical load case for the design of the slab.

1. BACKGROUND: DESIGN & CONSTRUCTION OPTIONS

For most buildings, ground floor slabs-on-grade are typically designed using empirical standards of practice that require little engineering effort and result in relatively thin and cost-effective slab sections. Analytical methods using nomographs are also available to designers that account for non-uniform loading conditions such as truck wheel loads and storage rack leg loads. Both empirical and analytical methods assume uniform subgrade stiffness where the soil is represented as linear–elastic springs (Figure 1a), commonly known as the “Winkler” subgrade model. Using methods outlined by the Portland Cement Association and others, the design of the floor slab includes applying simulated loads to the slab and evaluating computed shear stresses and bending moments. Resulting designs can include slabs constructed from plain concrete and concrete reinforced with conventional rebar or post-tensioned strands. The design typically is based on an uncracked section and is focused on limiting the concrete tensile stress to a value that is much less than the concrete modulus of rupture or flexural cracking stress. The concrete modulus of rupture \( f_{cr} \) is normally taken as:

\[
f_{cr} = 9\sqrt{f'_{c}} \quad \text{Eq. 1.}
\]

where \( f'_{c} \) is the concrete compressive strength (psi). A factor of safety of 1.7 is normally used in the design of a slab-on-grade. Conventional slabs-on-grade are often four to six-inches thick and are relatively inexpensive to construct.
When floor slabs are to be placed on undocumented fills, organic soils, and other compressible materials, and excessive settlement is intolerable, the slab design options usually consist of one of three choices:

1. Maintain the relatively thin concrete slab-on-grade design philosophy, but only if the unsuitable soils are excavated and recompacted or replaced with more competent materials (Figure 1b). The floor slabs are then analyzed with the "Winkler" subgrade method previously discussed, which results in slab sections comparable to those on suitable soils. The added cost of this option is related to the cost of the earthwork, costs that can quickly become prohibitive at sites with deep cuts, contaminated soils, high groundwater, or adjacent structures that must be protected or underpinned.

2. Install piles or drilled concrete caissons to support a structural slab (i.e. a slab that is structurally designed and reinforced to be able to span between installed deep foundations). A pile-supported structural slab (Figure 1c) alone can cost as much or more than the excavation and replacement option. Because of the very high stiffness ratio between the piles and the natural soils, the piles are assumed to resist the entire slab load and the slab must be capable of structurally spanning between the pile supports. In this case, the stiffness and support of the in-situ soil between the piles is completely disregarded in the analysis.

3. Install Geopier Rammed Aggregate Pier® (RAP) elements to reinforce the compressible soils and allow for the construction of a relatively thin floor slab (Figure 1d). The piers are installed to reinforce the poor soils at a pier spacing that typically ranges between 8 and 15 feet on-center. Because the RAP elements are stiffer than the surrounding soil, they attract floor slab loads forming a non-uniform support condition. Similar to pile-supported structural slabs, the floors must be designed to resist shear stresses and bending moments that develop as the applied loads attempt to span to the stiffer supports.

However, these stresses are significantly lower than those for pile-supported structural slabs because of the reduced stiffness ratio.

This Technical Bulletin focuses on the slab design approach for the RAP design option. The construction of RAP reinforcing elements is well described in the literature (Lawton and Fox 1994, Fox and Cowell 1998, Wissmann et al. 2000). Unique to the process is the use of direct vertical ramming action on thin lifts of placed aggregate, resulting in piers of high strength and stiffness (Wissmann et al. 2001).

The RAP technique results in a subgrade that has a non-uniform stiffness distribution: high stiffness at the RAP elements and low stiffness in the areas supported by the matrix soil between the piers. Therefore, the slab experiences shear and bending moment demands between those experienced by a structural slab and a slab-on-grade. Structural finite element analyses may be used to compute induced slab bending stresses and shear stresses. Design variables used in the finite element analysis include imposed uniformly-distributed area load, concrete compressive strength, RAP stiffness, in-situ soil stiffness, RAP spacing, and slab thickness.
Figure 1a. "Winkler" Beam Method

Figure 1b. Removal & Replacement of Compressible Soil

Figure 1c. Pile-supported Slab

Figure 1d. RAP Supported Slab
2. NUMERICAL ANALYSES

To understand the development of shear stresses and bending moments in RAP-supported floor slabs, a suite of structural finite element analyses was performed by KPFF structural engineers, John P. Miller, P.E., S.E., Principal and Jason N. Richards, P.E., S.E., Associate. The analyses considered the response of the slab to uniformly-distributed loading conditions and accounted for subgrade support by using stiff springs at the RAP locations and relatively soft springs to represent the matrix soil between the piers. The analyses were performed for variations of:

- Applied uniform floor slab loading pressure (w, expressed in psf),
- Concrete compressive strength (f’c, expressed in psi) and corresponding stiffness characteristics,
- RAP spring stiffness (k\_s, expressed in psi/in),
- Matrix soil spring stiffness (k\_m, expressed in psi/in),
- RAP spacing (L, expressed in feet),
- Floor slab thickness (t, expressed in inches).

The results of each analysis were used to compare the computed bottom fiber tensile stresses against allowable values to establish the allowable value of applied slab pressure for the modeled slab geometry and spring support conditions. These results were used to evaluate the maximum allowable uniformly-distributed load (prior to the development of limiting concrete tensile stress) for each value of normalized slab thickness (t/L). For simplicity, the analyses neglected stresses induced by concrete shrinkage and slab deformations, factors thought to be mitigated through the use of construction joints as described in section 2.1.

2.1 FINITE ELEMENT MODEL

A typical bay for a building with RAP foundation and floor slab support is shown in Figure 2. The piers are evenly spaced between the column bays with pier spacing determined from the characteristics of the matrix soils, floor slab loading, thickness of the floor slab, and slab construction joint spacing. The piers are commonly located directly underneath the construction joints where the joint may transfer shear stresses but not bending moments. The hatched area shown in Figure 3 indicates the extents of the finite element model used in this study, bounded on two sides by slab joints and on the opposite sides by lines of symmetry.

RAM Concept software (RAM International 2005) was used to perform the finite element simulations. To model the response of the slab, hybrid shell elements that can accommodate in-plane axial and shearing stresses as well as out-of-plane bending and shearing stresses were modeled. A concrete 28-day compressive strength (f’c) of 4000 psi was used in the analyses. The RAP spacing was varied from 8 feet to 16 feet on-center in two-foot increments. Figure 3 shows the finite element mesh used for the study.
2.2 SUBGRADE SUPPORT

Linear-elastic springs were used to represent subgrade support. Stiff springs \((k_s)\) were used to represent the 30-inch diameter RAP elements and relatively soft springs \((k_m)\) were used to represent the unimproved matrix soil response. A constant RAP spring stiffness \((k_s)\) value of 150 pci and matrix soil stiffness \((k_m)\) values ranging from 5 pci to 30 pci were used. The ratio of the spring constants is denoted by the stiffness ratio \((R_s = \frac{k_s}{k_m})\) and is a key determinant in the development of slab bending stresses (i.e. a stiffness ratio of infinity would result in a two-way structural slab design shown in Figure 1c; a stiffness ratio of unity would result in a conventional slab-on-grade design shown in Figure 1a). Table 1 presents a summary of stiffness constants and stiffness ratios used in the analyses.

The installation of the RAP elements increases the lateral stresses in the matrix soil which results in improved stiffness characteristics (Handy 2001). This soil improvement results in a transition from the high stiffness piers to the matrix soil elements. The stiffness transition function that was used in the analyses was taken from the results of plate load tests performed by researchers at Iowa State University (White 2004).

### Table 1
Range of parameter values considered in this study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values considered in this study</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAP center to center spacing (ft)</td>
<td>8, 10, 12, 14, and 16</td>
</tr>
<tr>
<td>RAP stiffness, (k_s) (pci)</td>
<td>150</td>
</tr>
<tr>
<td>Stiffness ratio, (R_s = \frac{k_s}{k_m})</td>
<td>5, 10, and 20</td>
</tr>
<tr>
<td>Slab thickness, (t) (in)</td>
<td>4, 6, 8, and 10</td>
</tr>
</tbody>
</table>

3. RESULTS

Figures 4 through 6 present the results of the numerical simulations for the 60 unique sets of geometry, subgrade support, and uniform loading conditions described in Table 1. The figures present contours of normalized thickness ratios \((t/L)\) required to limit the tensile stress demands imposed by normalized slab pressures \((w/f'c)\) to within allowable values. The contours shown on the figures were developed for pier spacing varying from 8 to 16 feet on-center. A required floor slab thickness value for various applications of uniform slab pressure may be estimated by using the following procedure:

1. Establish the appropriate pier to matrix soil stiffness ratio for the project site. The stiffness modulus of the RAP element \((k_s)\) is typically verified with a site-specific modulus test.
performed in accordance with procedures described in Fox and Cowell 1998. The matrix soil stiffness modulus ($k_m$) is obtained by computing the settlement of the unreinforced matrix soils in response to the floor slab pressure, where $k_m$ is the ratio of applied pressure to computed deflection. Note that values of $k_m$ computed using this procedure can result in values that are significantly lower than $k_m$ values often recommended in the literature for uniformly-supported floor slabs subjected to moving point loads.

2. Establish the normalized loading parameter value ($w/f'_c$) for the project. Include the weight of the slab when determining the floor slab pressure, $w$.

3. Select a RAP element spacing.

4. For the computed normalized loading parameter and selected RAP spacing, use Figures 4 through 6, as appropriate, to find the normalized required floor slab thickness ($t/L$) value. Should the input value for $w/f'_c$ result in a solution to the left of the dashed line shown in the figures, a minimum slab thickness of four inches should be used.

5. Estimate the required floor slab thickness ($t$) in inches to appropriately resist the induced tensile stresses by multiplying the normalized floor slab thickness value ($t/L$) by the RAP center-to-center spacing.

When using the design charts shown in Figures 4 through 6, it should be recognized that the results of the numerical analyses are subject to limitations. The computed values of tensile stress in the floor slab-on-grade are developed for uniform loading conditions only; other loading conditions and loading patterns, such as concentrated point loads, line loads, and “hopscotch” loading patterns, will result in different tensile stress values that may be more critical to acceptable slab performance. The modeled floor slabs included the assumption that a construction joint, which cannot transfer bending moments, is placed over the piers. Floor slabs with differing joint orientations should be evaluated separately. The models also exclude the presence of engineered fill between the tops of the RAP elements and the bottom of the floor slabs, which would improve the uniformity of the slab support characteristics. The analyses are based on measured subgrade support conditions for Rammed Aggregate Pier elements. These results should not be extended to other types of ground improvement because of variations in stiffness ratios and differences in the radial soil stiffness function resulting from differences in installation procedures.
Figure 4.
Normalized Thickness Required for Stiffness Ratio \( \frac{k_e}{k_m} \) of 5
Figure 5.
Normalized Thickness Required for Stiffness Ratio \( \frac{k_y}{k_m} \) of 10
Figure 6.
Normalized Thickness Required for Stiffness Ratio \( \frac{k_g}{k_m} \) of 20
4. CONCLUSION

Rammed Aggregate Pier® soil reinforcing elements are commonly used to support relatively thin concrete slabs-on-grade with light to heavy slab loads. The design of the floor slabs should consider the non-uniform support conditions offered by the stiff RAP reinforcing elements in contrast with the relatively soft matrix soil between the piers. These non-uniform support conditions may be studied using structural finite element analyses. The results of structural numerical analyses performed to compute the response of uniformly-loaded concrete floor slabs supported by RAP elements for variable pier to matrix soil stiffness ratio values, variable pier spacing, and ranges of uniformly applied floor slab pressures are presented in Figures 4 through 6 herein. These results provide estimated floor slab thicknesses (t), which can adequately resist the applied pressures without developing tensile stresses that exceed allowable capacity. The design charts presented herein are for uniform loading conditions only; project-specific analyses should be performed for other loading conditions.

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REFERENCES


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