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LIQUEFACTION MITIGATION OF THREE PROJECTS IN CALIFORNIA

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ABSTRACT

Ground displacements resulting from earthquake-induced soil liquefaction and dynamic densification can cause moderate to severe structural damage during and after an earthquake. Geotechnical construction methods of mitigating these potential ground displacements include mass excavation and replacement with engineered fill, ground improvement such as soil mixing, jet grouting, compaction piers, vibro compaction, vibro stone columns, and deep dynamic compaction, or deep foundations such as driven piles. The ground improvement methods rely on altering the soil properties to resist the seismically-induced shear stresses and soil grain redistribution while deep foundation methods bypass liquefiable soil deposits to found in deeper competent soil or rock.

This paper presents an advancement in displacement ground improvement methods used to control soil liquefaction potential by driving highly compacted aggregate into the soil deposit. The ground improvement is accomplished by driving a pipe mandrel to displace the soil mass, backfilling the cavity with select aggregate, and compacting the aggregate in controlled lifts utilizing vertical, vibratory driven methods to further displace and densify the soil deposit while creating a dense Rammed Aggregate Pier®. Specifically the ground improvement method 1) reinforces the soil deposit to resist and re-distribute seismic shear stresses, 2) increases the density and horizontal stress of the surrounding soil, and 3) provides a gravel drain to enhance dissipation of seismically-induced excess pore water pressure in the soil. Several projects performed in California, in areas of high seismic activity, have been tested for the resulting shear reinforcement effects and increased density effects manifested by this advanced method of construction. These projects and their resulting field test results are presented and discussed.

INTRODUCTION

One of the most dramatic causes of damage to structures during earthquakes is the occurrence of liquefaction in saturated sand deposits. These damages are caused by deformation or instability of soil masses ranging from mildly sloping ground to embankment slopes, increased lateral pressures against retaining structures, loss of bearing support for shallow or deep foundations, loss of lateral support for embedded structures or piles, lurching of level ground, flotation of buried conduits or tunnels, and settlement caused by reconsolidation of the liquefied soils (Idriss and Boulanger, 2008). Controlling these potential damages is the responsibility of geotechnical engineers and the contractors who implement the liquefaction mitigation schemes.

Mass excavation methods effectively reduce the dynamic settlement potential by replacing low density soil with high density engineered fill. However, this method is limited to relatively shallow depths and is difficult to accomplish in high groundwater environments. Ground improvement methods have become increasingly more common as effective methods to control earthquake-induced displacements. Ground improvement methods rely on altering the engineering

properties of the soil mass to resist the seismically-induced shear stresses often permitting the use of traditional structural engineering systems. Deep foundation methods bypass the liquefiable soil to found in deeper competent soil or rock and often require more costly structural engineering systems. This paper is focused on the Rammed Aggregate Pier ground improvement methods.

RAMMED AGGREGATE PIER SYSTEMS

The use of Rammed Aggregate Pier® (RAP) Systems is well documented in the literature over the last 20 years (Fox 1994, Majchrzak et. al, 2004, Farrell et. al, 2008). RAP Systems have been used in the United States, Canada, South America, Europe, and Asia to support school and hospital buildings, parking structures, water and wastewater treatment plants, large diameter water and oil/fuel tanks, retaining walls, and railroad/highway embankments. The engineering principles of the RAP ground improvement system are 1) installing a very dense, stiff, RAP into the matrix soil, 2) increasing the density or stiffness of the surrounding matrix soil, and 3) increasing the horizontal stress in the surrounding matrix soil. RAP construction creates a composite RAP/soil matrix with

increased strength and stiffness properties. The wide acceptance and use of the RAP system is evidenced by over 2,500 completed projects worldwide and locally in California by recent Federal, State, and Local City contract documents specifying RAP foundation support for their structures such as the Coast Guard Command Center at Yerba Buena Island in San Francisco, the Recreation and Wellness Center at Sacramento State University, the City of West Sacramento Community Center and the City of Brentwood Civic Center.

The types of RAP construction that are common today include replacement RAP construction known as Geopier® and displacement RAP construction known as Impact® and Rampact® pier. The displacement RAP systems are commonly used at sites with loose soil and high ground water or at sites with contaminated soil. The replacement RAP system is commonly used when the soil can be easily drilled with little to no pier casing.

Both RAP systems provide uplift resistance on the order of 25 to 100 kips ASD with the addition of a structurally designed steel anchor assembly into the pier aggregate. In California, the structural steel anchor typically consists of two bars for Impact piers or four bars for Geopier RAPs (Farrell et al. 2008).. Figure 1 shows an uplift RAP anchor assembly.

Detailed presentations and discussions of RAP design and construction methods for settlement control, uplift and lateral resistance are presented and comparisons of calculated to measured settlements are discussed in the literature for many projects in the United States and California (Farrell et al., 2004, 2008; Hoevelkamp and FitzPatrick, 2005). This paper discusses and presents results of the Impact pier RAP construction as an effective ground improvement tool to mitigate the potential liquefaction of loose soil deposits.



Fig. 1 RAP Uplift Bottom Assembly and Top Anchor

DESCRIPTION OF IMPACT PIER SYSTEM

Vibratory modification of soil deposits to mitigate liquefaction potential and post-liquefaction consolidation is well documented in the literature (Mitchell 2008, Idriss and Boulanger 2008). The Impact pier system is an improvement to the vibratory rod and vibratory replacement stone column methods described by Mitchell, Idriss and Boulanger. The



Fig. 2: Impact pier equipment with standard rammer head

Impact system is a vibratory driven, dry bottom feed, displacement method that utilizes dry installation techniques thereby eliminating the potential of generating significant spoil and water handling and saturating of clay soil layers.

This RAP system increases the density of the matrix soil by vibratory driving of a specially designed pipe mandrel with an advanced rammer head into the ground with the assistance of heavy equipment vertical crowd force see Fig. 2. After the mandrel is driven to the design elevation, select aggregate is loaded into the pipe mandrel to load rock to the bottom of the displaced soil. The mandrel is raised to charge the resulting displaced hole with rock and then the mandrel and advanced rammer head is driven back down to increase the density and to displace and compact thin lifts of the aggregate into the matrix soil, see Fig. 3. Densification is achieved during successive RAP drive strokes using vertical crowd force applied by the machine and vibratory impact ramming energy delivered by a high-frequency, vibratory, pile hammer. The advancements of this method include 1) the installation by dry displacement of the soil, backfill with select aggregate, 2) vertical compaction of aggregate creating a stiffened aggregate pier inclusion, 3) compaction of the aggregate utilizing vertical hydraulic crowd force and high frequency hammer forces which displaces aggregate laterally into the soil and further expands the displaced cavity stiffening the matrix soil and increases the density of loose sands.

This RAP method produces improved aggregate and soil stiffness compared to vibro stone column methods where horizontal vibrations, that are produced by eccentric weights in a vibrating probe, are used to vibrate the stone (FHWA, 1983 a and b). This RAP method deposits a controlled volume of aggregate in every rammed lift coupled with mechanical vertical ramming of each aggregate lift.

Impact piers can be constructed to diameters ranging from 18 to 30 inches (457 to 610 mm) and to depths of 50 feet (16 m). Increasing RAP drive strokes results in lateral improvement zones to distances of 3 diameters from the center of the RAP. The ramming equipment consists of a 50 to 74 ton (445 to 658 kN) piling rig equipped with a 75 to 150 ton (667 to 1,335 kN) vibratory piling hammer, pipe mandrel, and an advanced expanded beveled rammer head as seen in Fig. 1.

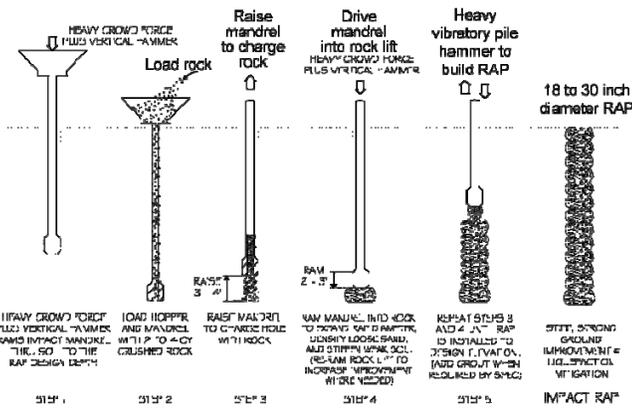


Fig. 3 Impact pier Method of Construction

RAP MITIGATION OF LIQUEFACTION POTENTIAL

Liquefaction potential of soil deposits is well understood, but the methods for mitigating its potential and effects are limited. The historical methods for engineering a solution for liquefaction has been to drive piles thru and below the liquefiable soil zones to found in deeper competent soil and bedrock layers. In the 1940's and 50's densification of loose sands was introduced in the industry and became well known and used widely in subsequent years. In recent years, the use of densification is still one of the most common specified methods for mitigating liquefaction potential.

Soil improvement by impact pier, vibro-rod and vibro-replacement stone columns provide four mechanisms in which they can mitigate liquefaction, these include 1) densification, 2) increases in lateral stress, 3) reinforcement of soil mass, and 4) improved drainage. However, reinforcement and drainage are often not accounted for in the protection and factor of safety estimates for liquefaction mitigation (Boulanger 2000). As ground improvement technologies, densification techniques have been the more familiar method of controlling liquefaction and the potential resulting ground settlement. Densification is typically effective in loose sand with a fines content less than about 20% passing the number 200 sieve (Idriss and Boulanger 2008).

The difficulty in estimating the improvement benefits of densification methods is usually left to the engineer and ground improvement contractor's experience and is commonly confirmed after construction of the densified soil by comparing pre-construction SPT or CPT tests to post construction SPT or CPT tests in between some improvement spacing. This method is reliable, but does not allow engineers and contractors to use engineering calculations to estimate construction spacing and thereby presents some risk for the engineer and contractor if the densification requirements are not met at an estimated and contracted improvement spacing.

The reinforcement of the soil mass, also know as the shear reinforcement method or shear stress redistribution method, gives the engineer and contractor a means to perform engineering calculations to estimate construction spacing prior

to after the fact testing. When taken into account, this method increases the factor of safety from the commonly used densification only methods and provides the needed improvement for those higher fines content soils that do not respond to densification techniques.

Shear Stress Re-distribution Method

In 1999, Kocaeli Earthquake (M=7.4) in Turkey revealed the use of high modulus columns in the reduction of seismic shear stresses, thereby reducing liquefaction potential of dirty sand deposits (Martin et al 2004). The vibratory vertical ramming installation of Impact RAPs in a loose, saturated sand deposit can potentially mitigate the risk of liquefaction by decreasing the seismic demand on the soil by redistributing the induced shear stresses from the sand to the high modulus rammed aggregate piers. The RAP modulus is measured by full scale load testing at the project site, see Fig. 4. The response of high modulus RAPs has been studied numerically to understand the shear and flexural behavior during seismic action.

The response of an aggregate pier foundation system during seismic loading was investigated by a comprehensive numerical model using FLAC (Girsang and Gutierrez, 2001; Girsang et al, 2004). The research was divided into three parts: 1) ground acceleration, 2) excess pore water pressure ratio, and 3) shear stress distribution in the soil matrix generated during seismic loading. Two earthquake time histories scaled to different maximum acceleration (pga) were used in the numerical modeling: the 1989 Loma Prieta earthquake (pga = 0.45g) and the 1988 Saguenay earthquake (pga = 0.05g). The results of the simulation showed that the stiff column/pier reduces the liquefaction potential due to stress concentration to the column/pier to the depths where it is installed; that pore pressures are generally lower for soils reinforced with aggregate pier than unreinforced soils as long as the applied shear stresses do not exceed the cyclic shear resistance of the aggregate materials; and that the maximum soil shear stresses are much smaller for reinforced soils than unreinforced soils.

Shear stress re-distribution was also evaluated in a 2-dimensional, total stress, plain strain Finite Element Analysis using Dynaflow (Prevost 2007) and it was found that RAPs and the immediate surrounding soils deform in a combination of shear and flexure. The percent contribution of shear versus flexural deformation of the column/pier varies with depth, with the column/pier deforming predominantly in flexure near the ground surface and predominantly in shear at depth. The percent contribution of each mode of deformation governs the redistribution of the shear stresses from the soil to the stiff column/pier and thereby the install RAP elements. The distribution of the shear stresses between the soil and RAP elements are quantified for site-specific properties based on soil conditions observed at each site (Green, Olgun, Wissmann, 2008).

The shear stress redistribution method is based on the higher elastic modulus of the RAP absorbing the seismic shear stresses in the native soil. Each of the case history projects described include full scale modulus load testing and pre and post-installation cpt testing to verify that liquefaction mitigation was achieved for the design earthquake. The modulus tests were performed to confirm the RAP stiffness modulus used to estimate shear reinforcement effects and the pre and post cpt tests were used to confirm the sand densification effects of RAP construction.

Modulus Test Configurations

Figure 4 shows a RAP modulus test section and a photo of the test set up. The test set up consists of a compression element, two uplift elements or reaction piers, and a reaction frame. The RAP is loaded to 150% of the calculated maximum top-of-pier stress (Majchrzak et al 2004). The load is applied against the reaction frame and resisted by the reaction piers. The modulus test measures the RAP stiffness used to estimate the reinforcement effects in the soil deposit. The modulus tests are performed to confirm that the piers are substantially stiffer than the surrounding matrix soil, often 10 to 30 times. The RAP modulus test results are presented for each project below.



Fig.5. Section and Photo of RAP Modulus Test Set-up

IMPACT PIER PROJECT DESCRIPTIONS

Three Impact pier projects from California are presented in this paper; the Moran Asian Gardens Parking Structure and Condominiums project in Westminster, CA; the Restaurant Depot Project in Oakland, CA; and the Iron House Waste Water Treatment Plant site in Oakley, CA. Pre- and post-installation CPT resistance, modulus tests, and measured settlements are presented.

The Moran Asian Gardens project consisted of an approximately 150,000 square feet (13,900 square meters) building footprint. The building structure consists of a two-story partial subterranean concrete parking structure with 4

stories of wood-frame construction above. The site is underlain by alluvial soil deposits with the large majority of the soil consisting of loose to medium dense clean sands and silty/clayey sands. Standard penetration blow counts generally range between 3 and 20 to depths of 30 feet across the site. Groundwater is present at 5 feet below the adjacent streets. Peak ground accelerations for the project reach $PGA=0.42$ g at a magnitude $M_w=6.9$. The project's major soil condition issues included static settlements of 2 to 3 inches plus liquefaction and post seismic settlements of up to 3 inches.

The Restaurant Depot project consisted of a 84,000 ft² (7,800 m²) building footprint. The building structure consists of a steel frame warehouse with heavy slab loads generated by stacking of food and equipment. The site is underlain by loose hydraulic sand fill over layered bay mud and sand lenses. Standard penetration blow counts range between 2 and 17 to depths of 35 feet. Groundwater is present at depths of 5 feet below pad elevation. Peak ground accelerations for the project reach $PGA=0.64$ g at a magnitude $M_w=6.72$. Impact piers were installed to a depth of 15 feet below the slab and 25 to 35 feet at footing locations. The typical RAP spacing was 7'-0" on center in an equilateral triangle pattern. Figure __ presents pre and post CPT results for RAPs installed to 35 feet improvement zone. The post CPT was located in the center of a triangular spacing. The project's major soil condition issues included static settlements of 2 inches plus liquefaction of the hydraulically placed sand fill and post seismic settlements of up to 3 inches.

The Ironhouse Waste Water Treatment Plant Expansion project in Oakley, CA consists of several waste water process tanks and equipment buildings in the Delta region. The site is underlain by loose sand over layers of bay mud then loose sand deposits to depths of 20 feet with overconsolidated clays at greater depth. Standard penetration blow counts range between 2 and 16 to the depths of 20 feet. Groundwater is present at depths of 1 to 2 feet below the pad elevation. Peak ground accelerations for the project reach $PGA=0.37$ g at a magnitude $M_w=6.5$. The project's major soil condition issues included liquefaction and post seismic settlements of up to 2 inches including static settlement issues of up to 4 inches.

CASE HISTORY RESULTS OF CONSTRUCTION

Each project presented here had loose soil conditions that would lead to static settlement under the proposed structural loads and to post liquefaction settlement after a major earthquake. Impact piers installed at these project sites extended to depths between 15 to 38 feet below the ground surface. At each site, the impact piers increased the overall soil stiffness with high modulus columns, increased the density of liquefiable sands, and provided a gravel drain path at the Westminster and ISD WWTP sites. Impact piers at the Restaurant Depot site were fully grouted below 15 feet to control cross aquifer groundwater migration due to petroleum contamination in the soil in the upper 15 feet.

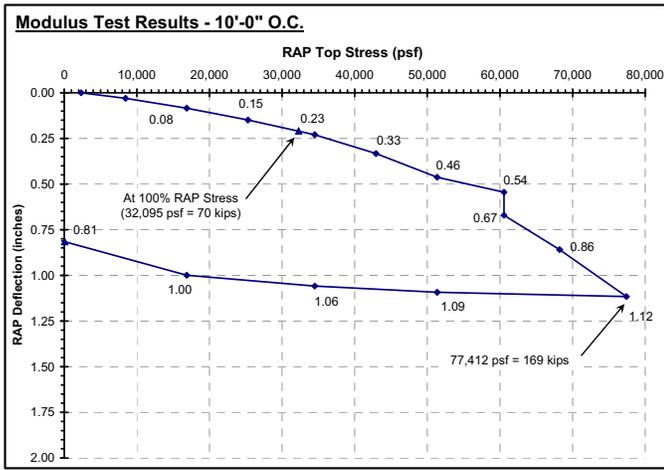


Fig. 6 Moran Asian 7' OC Modulus Test

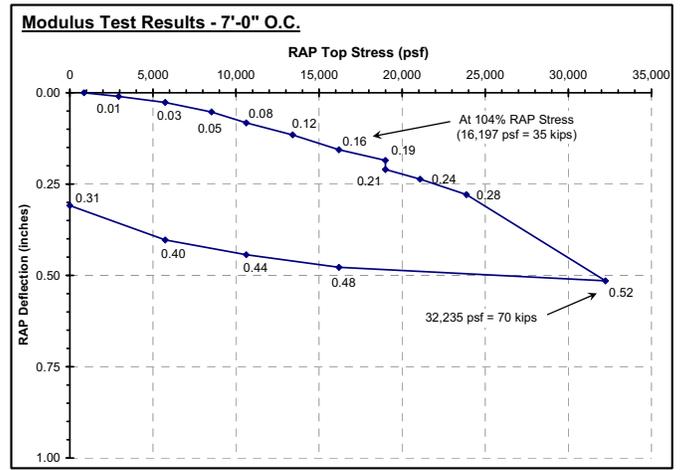


Fig. 8 Rest. Depot 7' OC Modulus Test

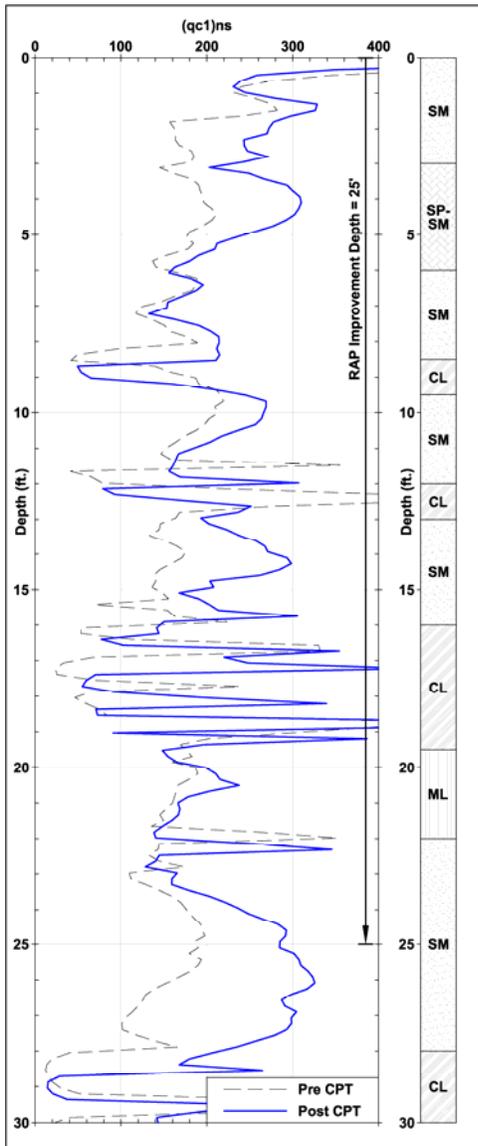


Fig. 7 Moran Asian 7' OC PRE/POST CPT

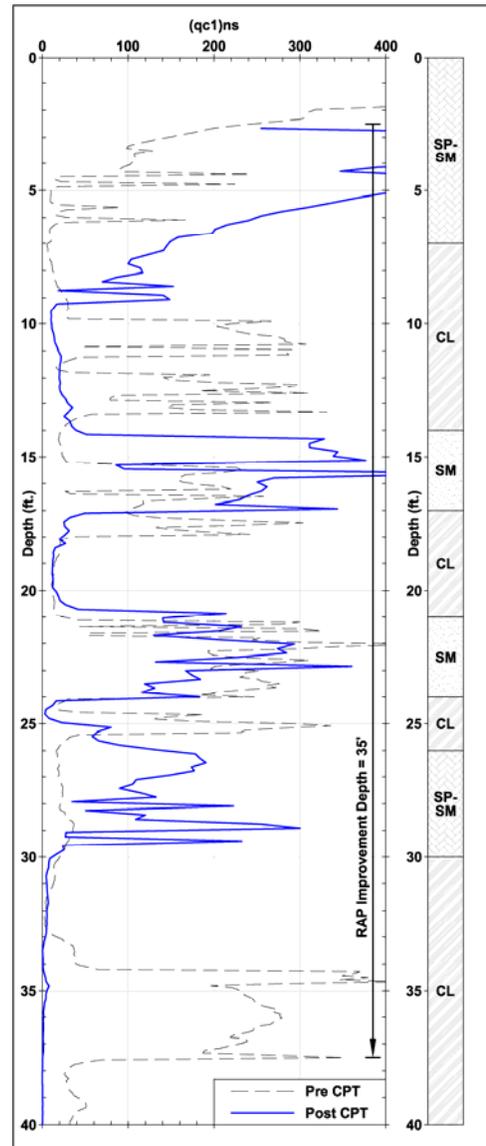


Fig. 9 Rest. Depot 7' OC PRE/POST CPT

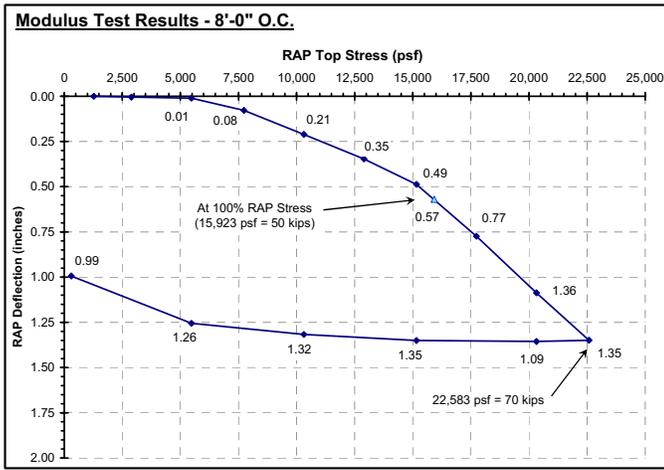


Fig. 10 ISD WWTP 8' OC Modulus Test

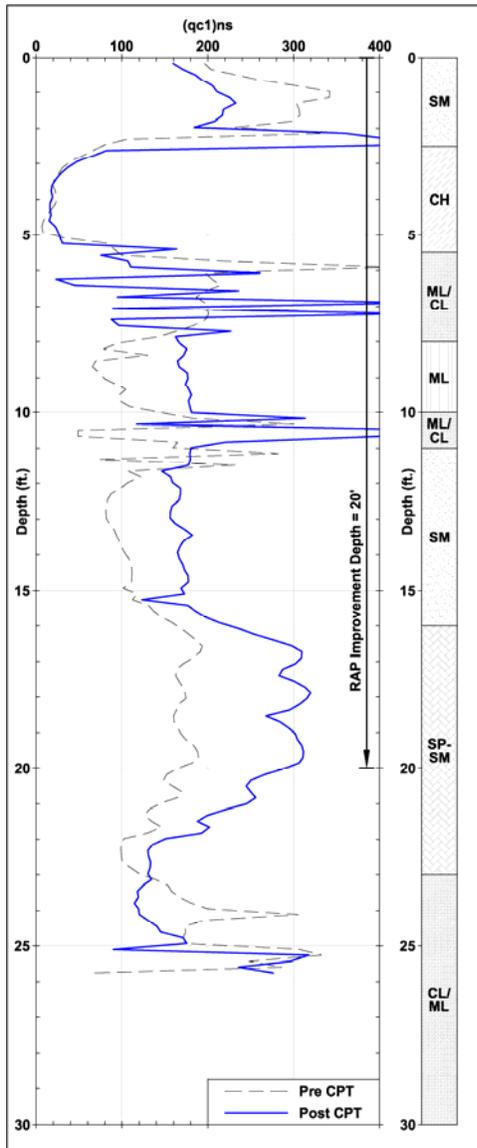


Fig. 11 ISD WWTP 8' OC PRE/POST CPT

Modulus Test and Post CPT Results

Figures 6, 8, and 10 present the results of the modulus tests. The purpose of the modulus test is to verify the RAP stiffness modulus (kg) used for settlement design calculations and to confirm the modulus used to estimate shear stress redistribution.

Figures 7, 9, and 11 present the results of the pre and post cpt tests. A review of post- to pre-treatment CPT tip resistance data indicated that significant densification effect was achieved in sandy soils at all three sites. Post-installation to pre-installation CPT ratios (referred to as improvement ratio below) ranged from approximately 1.3 to over 4 within the depth of treatment. Improvement ratios in shallow soils (in the upper 5 to 7 feet) ranged from approximately 1.63 to greater than 4, indicating that the application of vertical ramming process was effective in treating shallow soils even when there is little vertical stress/confinement. Further, improvement ratios within approximately 3 to 4 feet below the tip of RAP drive depth ranged from approximately 1.4 to 2.73.

SUMMARY AND CONCLUSIONS

Published data (Figure 80 on page 113 of Idriss and Boulanger, 2008) show that soils with a CRR value less than 0.5 and a measured CPT tip resistance, $(qc1)ns$, greater than 175 would fall within the non-liquefiable zone. It is noted that a review of Figures 7, 9, and 11 shows that a majority of the post-CPT tip resistance is greater than 175 that further confirmed the effectiveness of densification by using the Impact RAP construction method.

The advanced Impact pier construction method, using heavy crowd force and vertical ramming during installation, both RAP construction methods result in expansion of the aggregate at the edge of the pier (cavity expansion). With the use of high frequency vibratory ramming, the adjacent soil at distances of 4 to 5 feet away from the RAP drive strokes exhibits increased density as shown by pre and post cone penetration testing (CPT). The combination of increased soil density, higher lateral stress, the stiff Impact pier RAP, and the undulated shape results in enhanced coupling of the RAP aggregate to the matrix soil providing an efficient mechanism for shear resistance in the matrix soil. This method of ground improvement has been shown as an effective tool in densifying clean sands, providing non-liquefiable stiffened inclusions in clean sands and dirty sands, in addition to the secondary benefits of increased lateral stress and drainage.

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Symbols used in order of appearance:

P	dead plus live load downward force on a footing
q	applied bearing pressure
A	area of the footing bottom
Q _g	load resisted by rammed aggregate pier

Q_s	load resisted by soil
q_g	top stress on rammed aggregate pier
A_g	area of rammed aggregate piers below footing
q_s	bearing stress on soil
A_s	area of soil below footing
s_{uz}	upper zone settlement
k_g	stiffness modulus of rammed aggregate pier
k_s	stiffness modulus of unimproved soil
R_s	stiffness ratio
R_a	area ratio
N_{spt}	standard penetration test blow counts
s_u	undrained shear strength
H_s	length of drilled shaft below footing bottom
H_{uz}	thickness of upper zone soil
H_t	thickness of total zone of stress influence
H_{lz}	thickness of lower zone soil
I_s	stress influence factor at mid-depth of lower zone
f_s	vertical rammed aggregate pier shaft resistance
s_h'	effective horizontal earth pressure
k_p	Rankine horizontal earth pressure coefficient
s_v'	vertical effective stress
ϕ'_s	effective soil friction angle
T_{ult}	ultimate uplift resistance
W_{pier}	weight of rammed aggregate pier
ϕ'_g	rammed aggregate pier friction angle